Automatically Assembling Frescos from Noisy Pairwise Fragment Matches

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Motivation

Wall paintings hidden in volcanic ash in Akrotiri (Santorini), Greece

Antelope Fresco

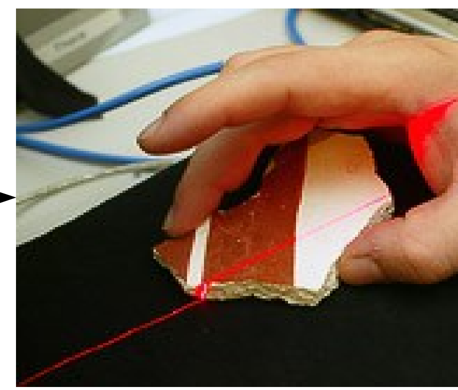
Part of a reconstructed fresco

https://artoffresco.com
https://www.santorini.com/archaeology/akrotiri.htm

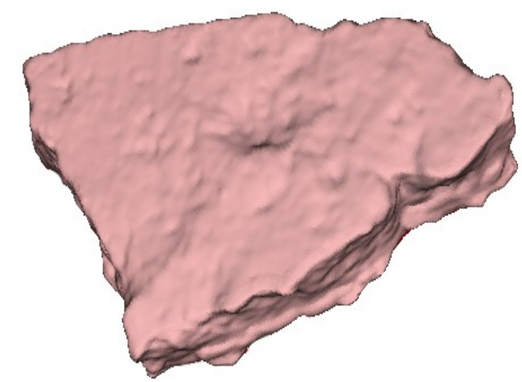
Introduction
Step 1: Scanning

- Computer-based reconstruction methods offer great potential in improving speed and accuracy of archaeological reconstruction efforts

Fragment collection



Surface Scanning



Scanned Fragment Mesh Representation of a Fresco

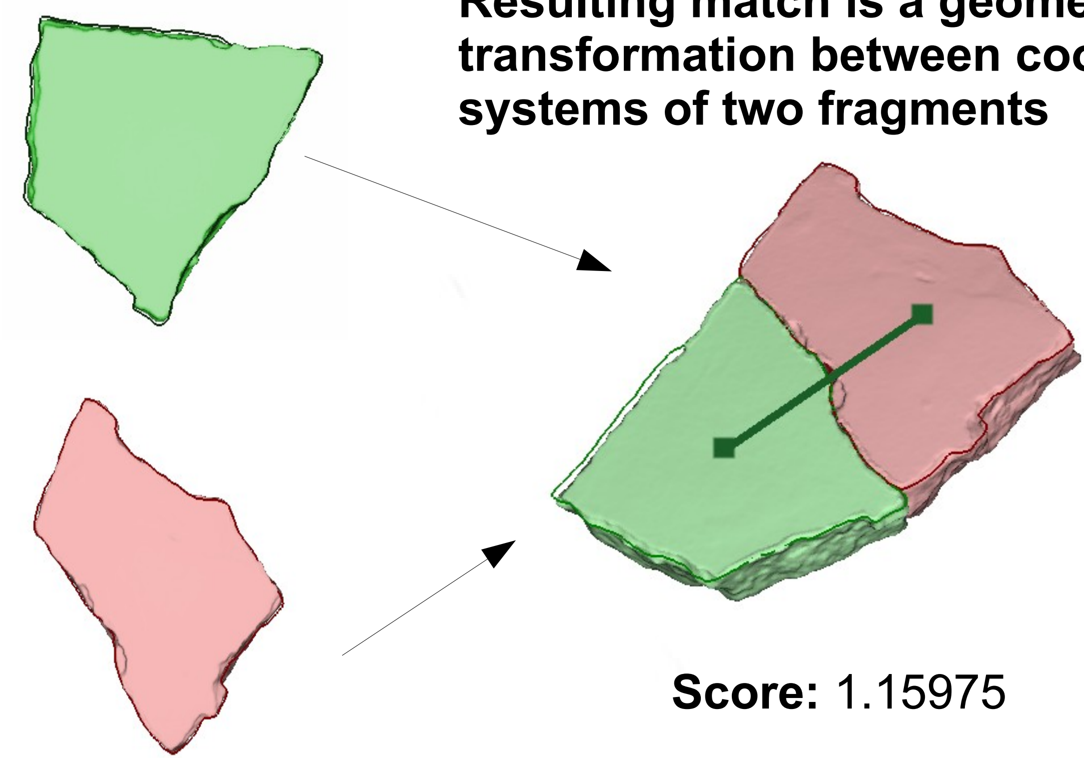
Benedict J. Brown, Corey Toler-Franklin, Diego Nehab, Michael Burns, David Dobkin, Andreas Vlachopoulos, Christos Doumas, Szymon Rusinkiewicz, and Tim Weyrich. A System for High-Volume Acquisition and Matching of Fresco Fragments: Reassembling Theran Wall Paintings. ACM Transactions on Graphics (Proc. SIGGRAPH) 27(3), August 2008.

Photo source: *http://www.princeton.edu/main/news/archive/S21/86/52G22/*

Introduction
Step 2: Matching

- Use an algorithm to find and rank potential pairwise matches between fragments

Resulting match is a geometric transformation between coordinate systems of two fragments



$$M = \begin{pmatrix} \cos\delta_{i,j} & -\sin\delta_{i,j} & t_1 \\ \sin\delta_{i,j} & \cos\delta_{i,j} & t_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$\delta_{i,j} = \theta_i - \theta_j$ - Angular offset

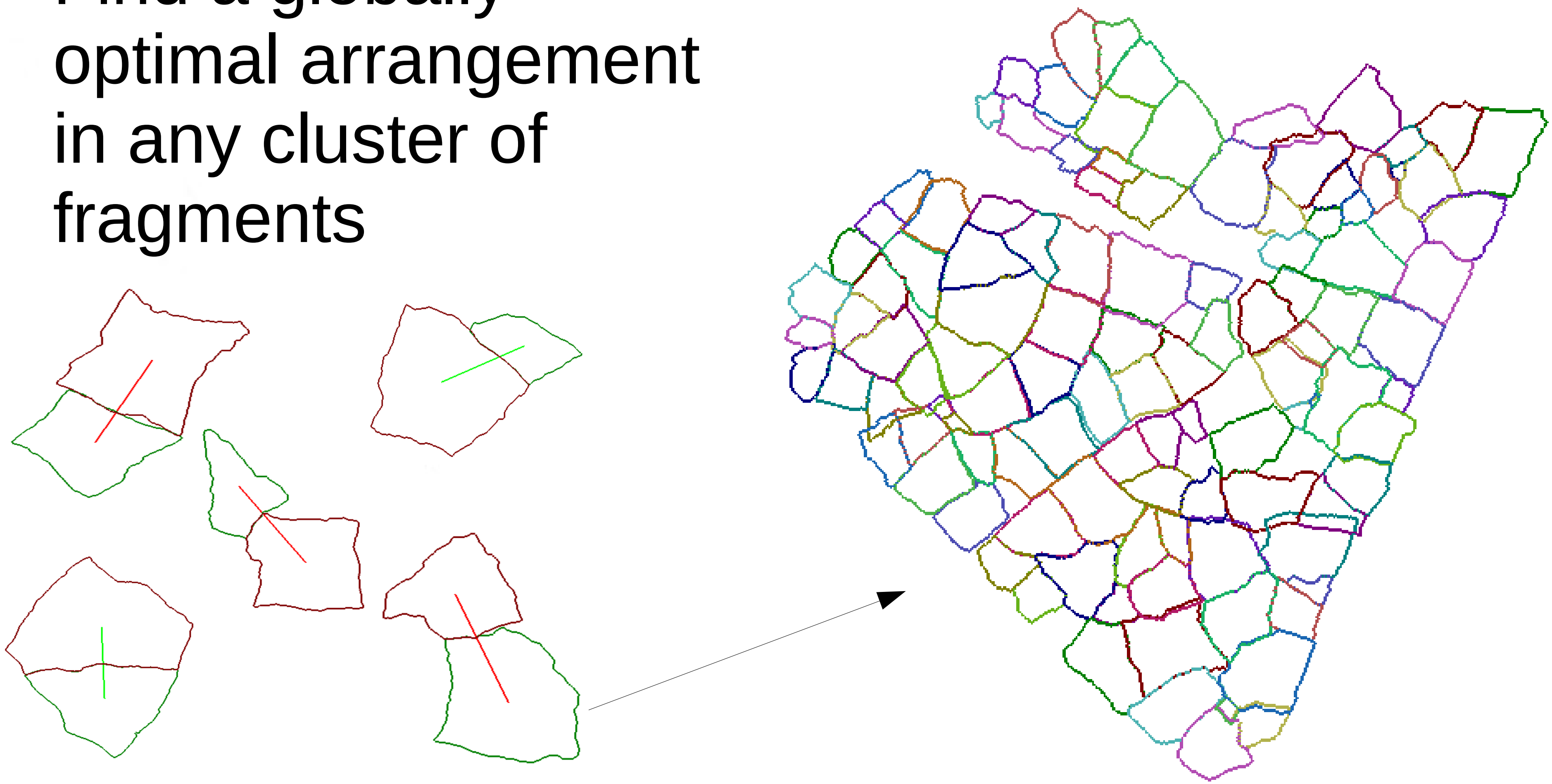
$T = (t_1, t_2)$ - Translational offset

Score: 1.15975

Thomas Funkhouser, Hijung Shin, Corey Toler-Franklin, Antonio García Castañeda, Benedict Brown, David Dobkin, Szymon Rusinkiewicz, and Tim Weyrich. Learning How to Match Fresco Fragments. Journal on Computing and Cultural Heritage 4(2), November 2011.

Introduction
Step 3: Global Assembly

- Find a globally optimal arrangement in any cluster of fragments

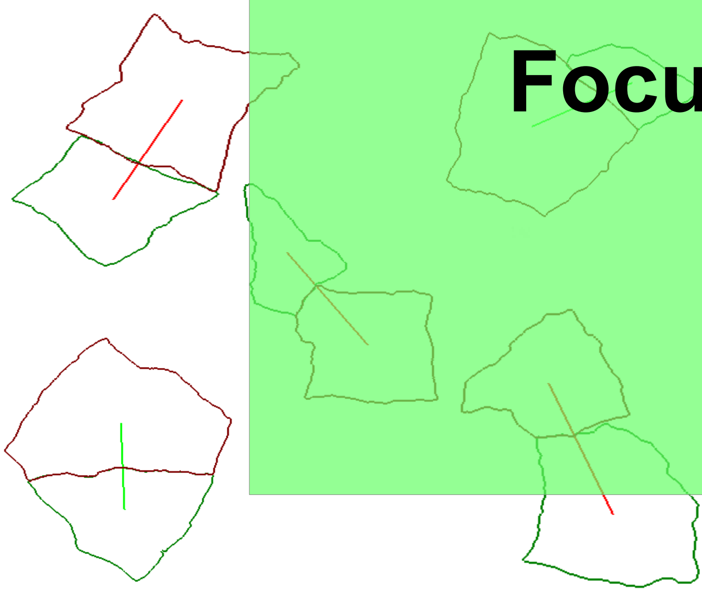


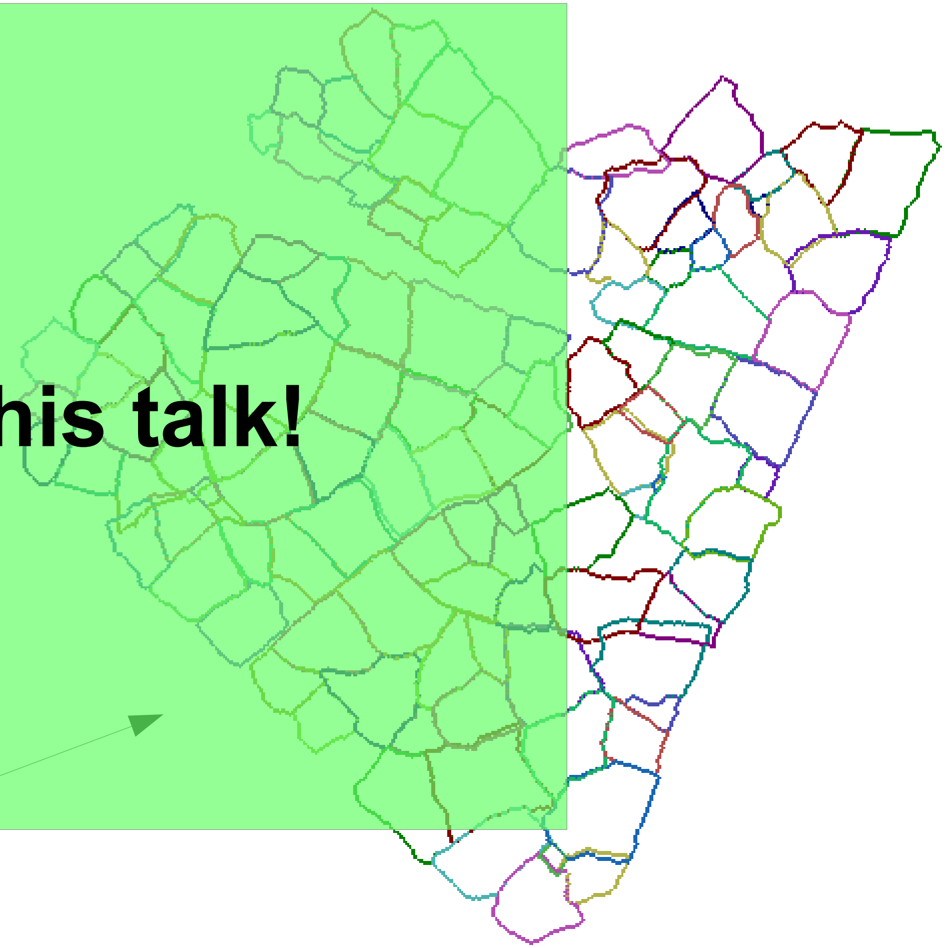
50K ranked matches, most are wrong

Introduction
Step 3: Global Assembly

- Find a globally optimal arrangement in any cluster of fragments

Focus of this talk!





50K ranked matches, most are wrong

Existing Assembly Strategies

- **Hierarchical clustering:**
  - clusters of fragments are merged by means of best candidate match
  - Fragment alignments are optimized in each iteration
  - process terminates when no good merges are possible
- **Issues:**
  - Optimization of alignments uses

    Least Squares (LS) minimization
  - LS does **not** deal with outlier matches well

Antonio Garcia Castaneda, Benedict Brown, Szymon Rusinkiewicz, Thomas Funkhouser, and Tim Weyrich. 2011. Global Consistency in the Automatic Assembly of Fragmented Artefacts. Intl. Symposium on Virtual Reality, Archaeology and Cultural Heritage (VAST) (Oct. 2011).

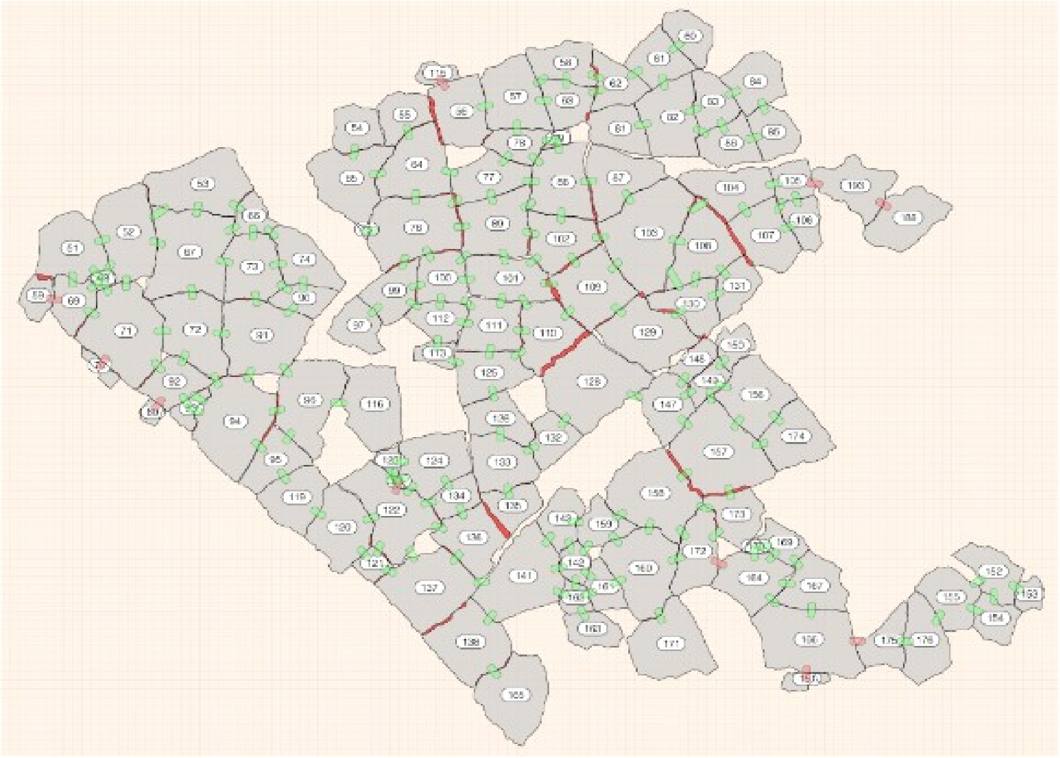
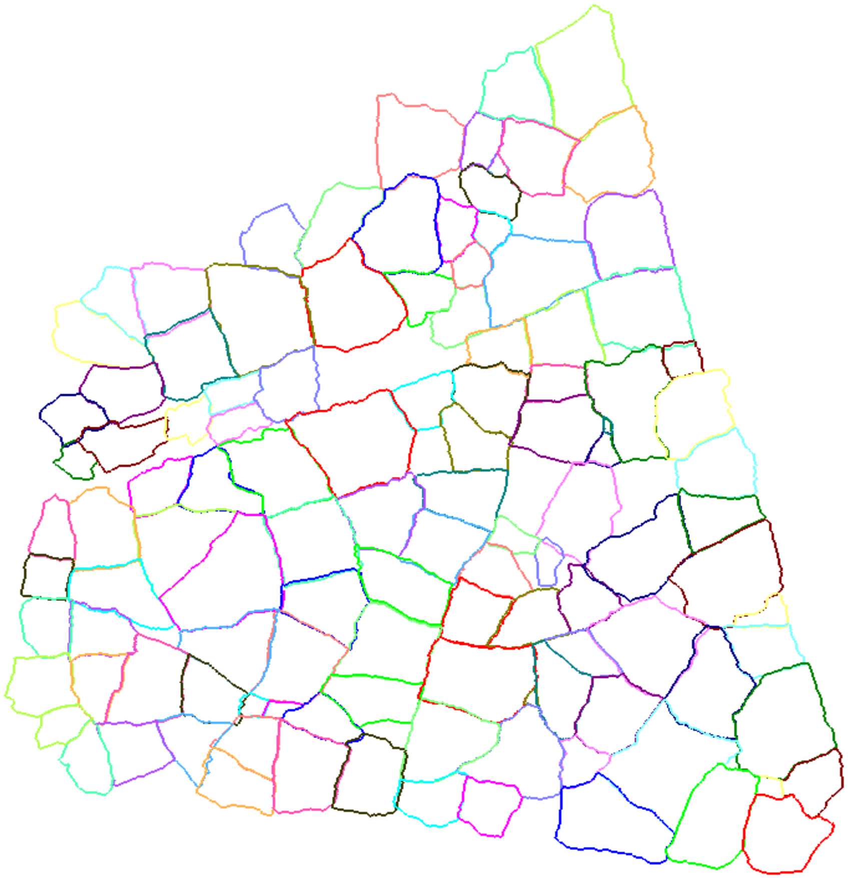
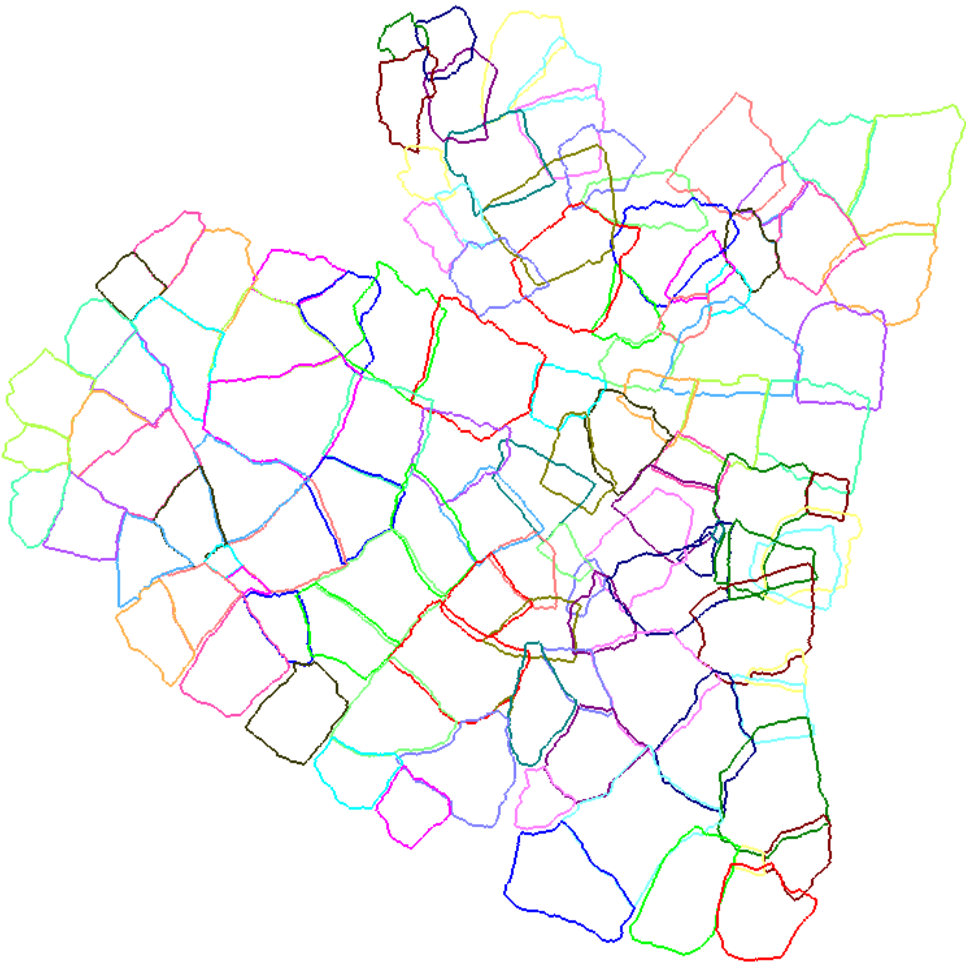


Figure: The assembled cluster contains 118 fragments and 188 matches, 10 of which are incorrect.

Problem With Least Squares



Original

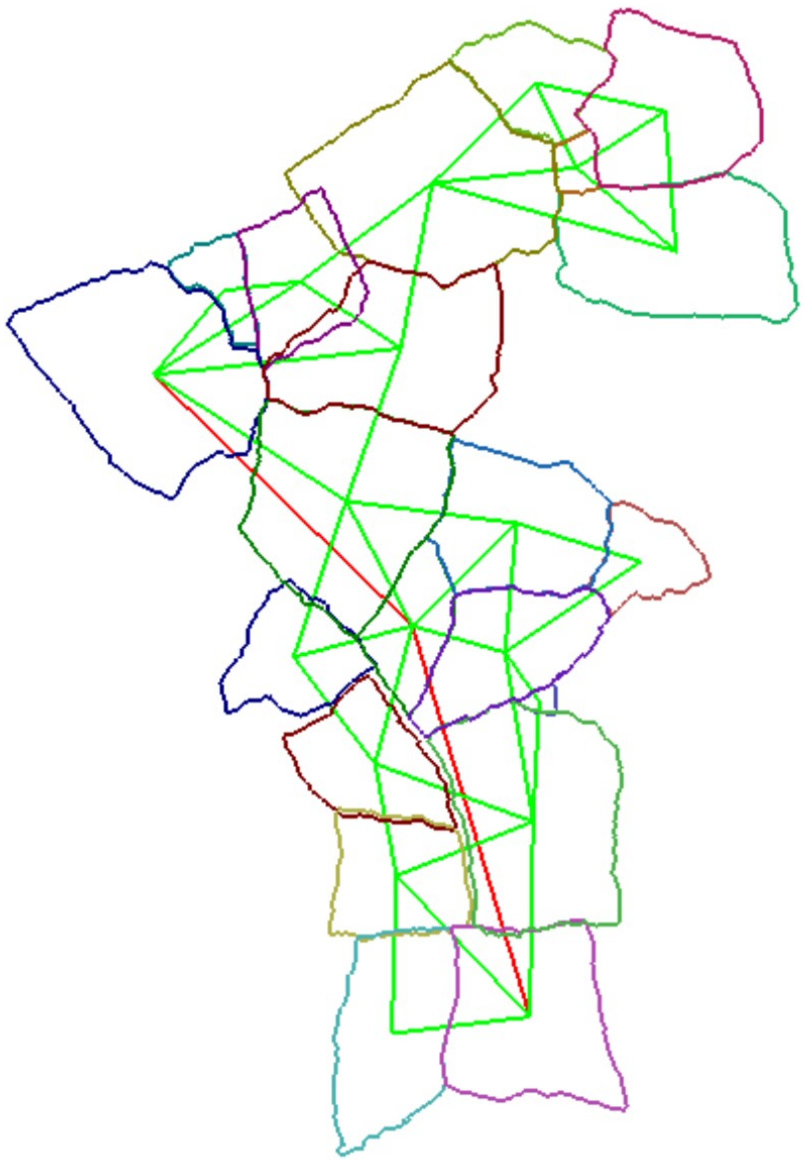


Least Squares (3 wrong matches)

This Project: Better Optimization

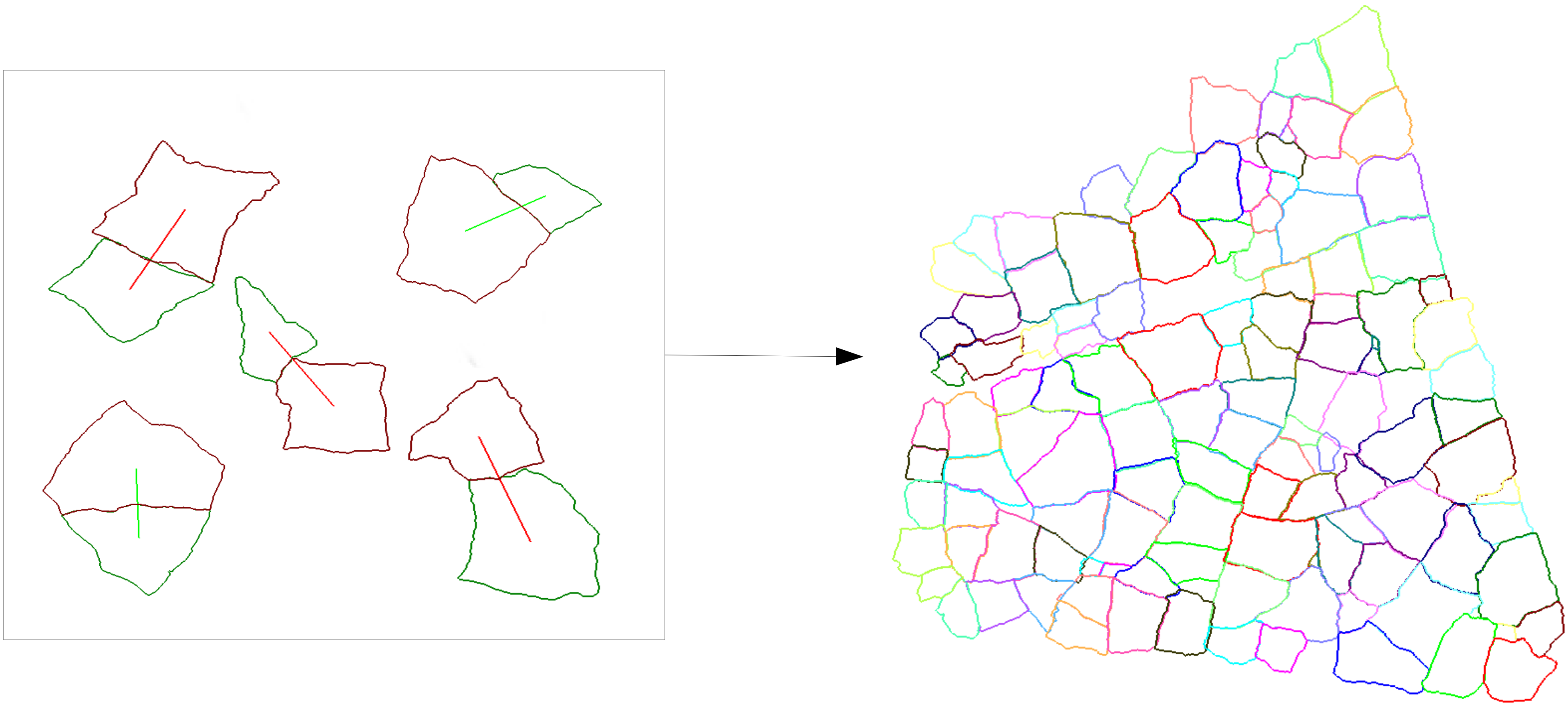
- The goal of this project is to solve the problem of previous optimization methods:

How to find globally optimal fragment transformations given a match set containing outliers?



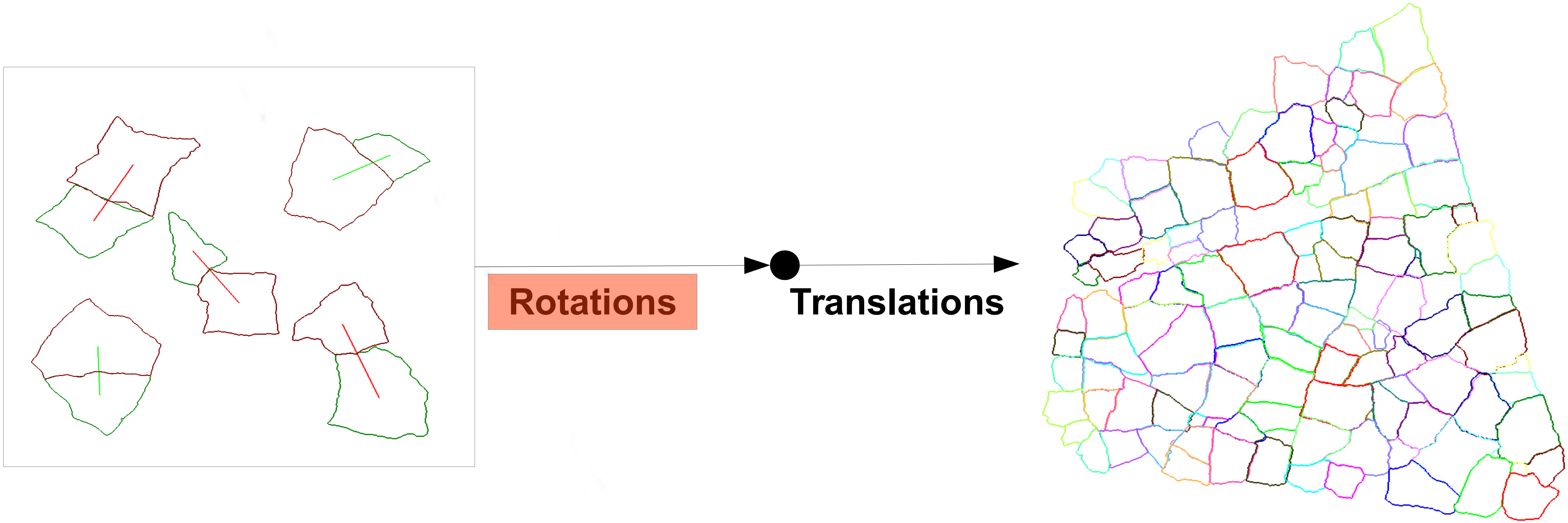
Approach

- Use a synchronization method robust to noise to recover globally optimal rotations



Synchronization

- Calculate rotations and translations separately



Solving for Rotations

- Find rotation angles for each fragment with respect to the global coordinate system
  - Find unknown angles

    $$\theta_1, \theta_2, \ldots, \theta_n \in [0, 2\pi)$$

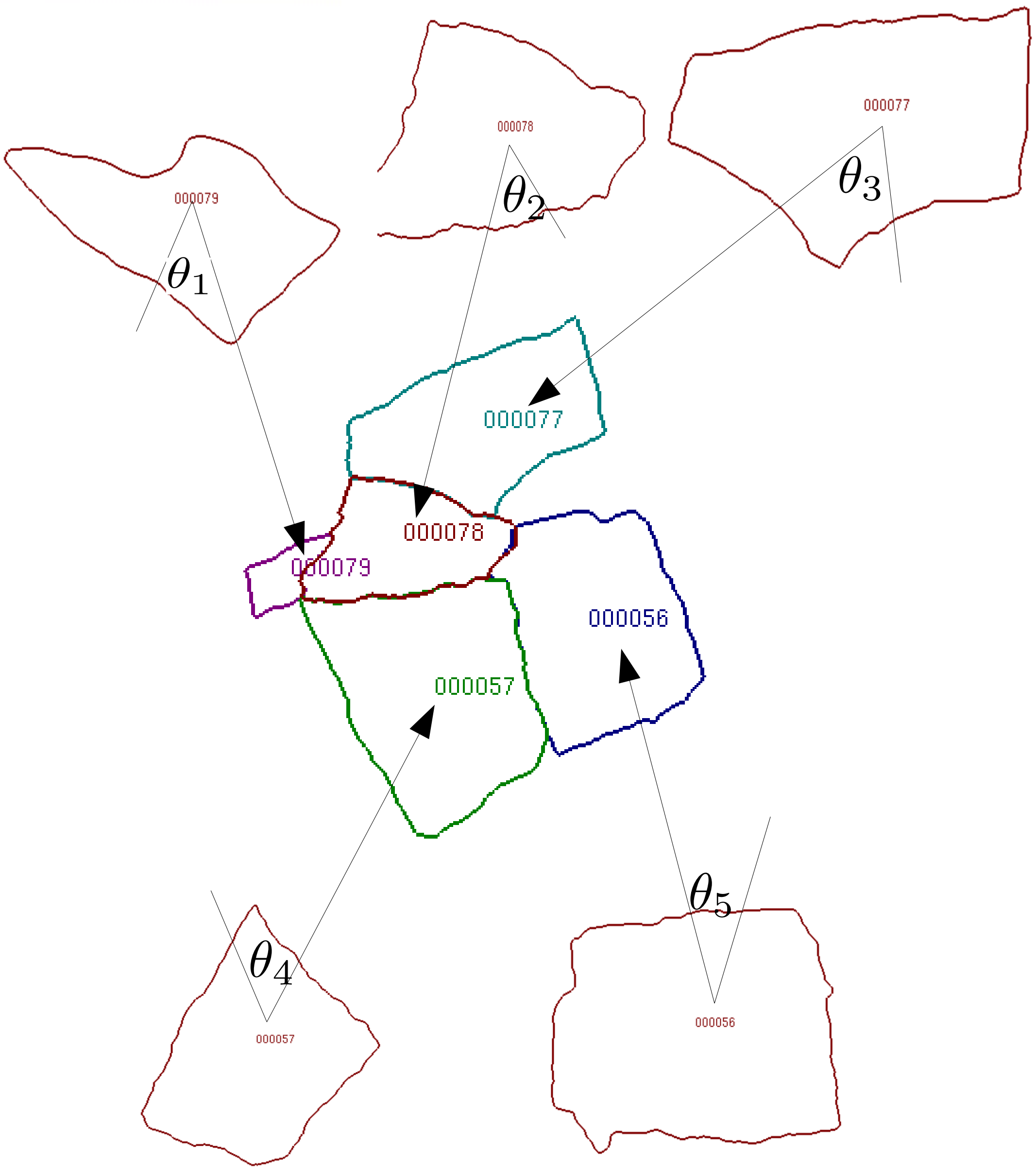
    from known pairwise matches

    $$\delta_{ij} = \theta_i - \theta_j.$$

Resulting set of equations:

$$\theta_i - \theta_j = \delta_{ij} \mod 2\pi$$

Solving the above is a non-linear problem



Solving for Rotations

- For each fragment i with (unknown) rotation angle θ_i, define variable $z_i = e^{i\theta_i}$
- Solving for rotations is equivalent to optimizing the objective:

$$\min_{\theta_1,\theta_2,\ldots,\theta_n} \sum_{i,j} \left(z_i - e^{i\delta_{ij}} z_j\right)^2$$

Subject to $|z_i| = 1$

Previous Method: Least Squares

Least squares cannot handle non-linear constraints.

$$\min_{\theta_1,\theta_2,\ldots,\theta_n} \sum_{i,j} \left(z_i - e^{i\delta_{ij}} z_j\right)^2$$

~~Subject to~~ $\cancel{|z_i| = 1}$

Solution: Normalized Eigenvalue Method (n*EVM*)

- Define: $$H = \begin{cases} e^{i\delta_{ij}} & \text{if match } \delta_{ij} \text{ is observed} \\ 0 & \text{otherwise} \end{cases}$$

- Normalize H by degree (number of matches per fragment)
- Compute top eigenvector of *H*
  - *Normalized entries of the resulting eigenvector represent angles (optimum respects angular constraint)*

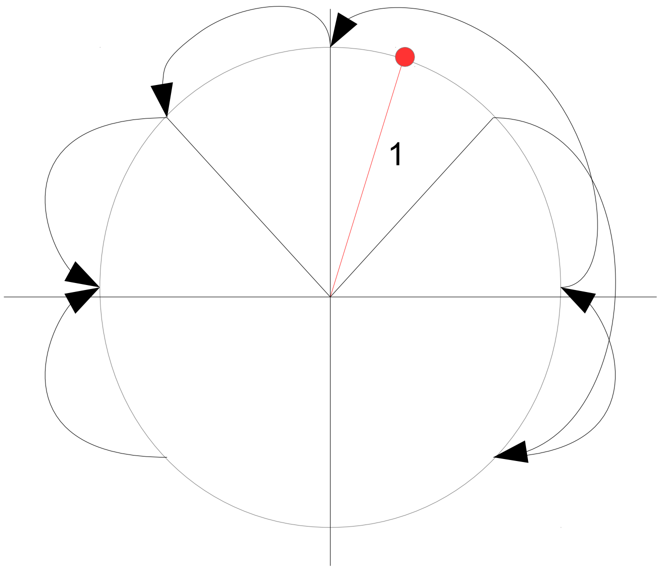
A. Singer, Angular Synchronization by Eigenvectors and Semidefinite Programming, Applied and Computational Harmonic Analysis, 30 (1), pp. 20-36 (2011).

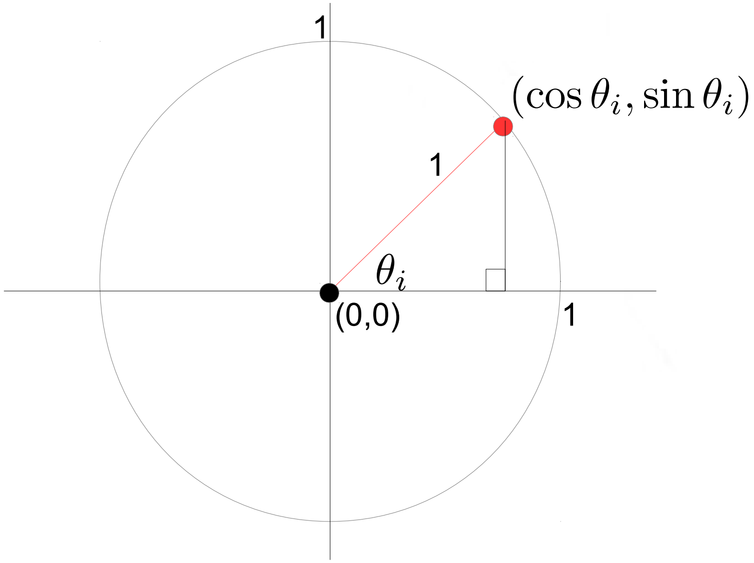
Robustness of nEVM

- nEVM incorporates the angular constraint

$$\cos^2\theta_i + \sin^2\theta_i = 1$$

  into the objective function





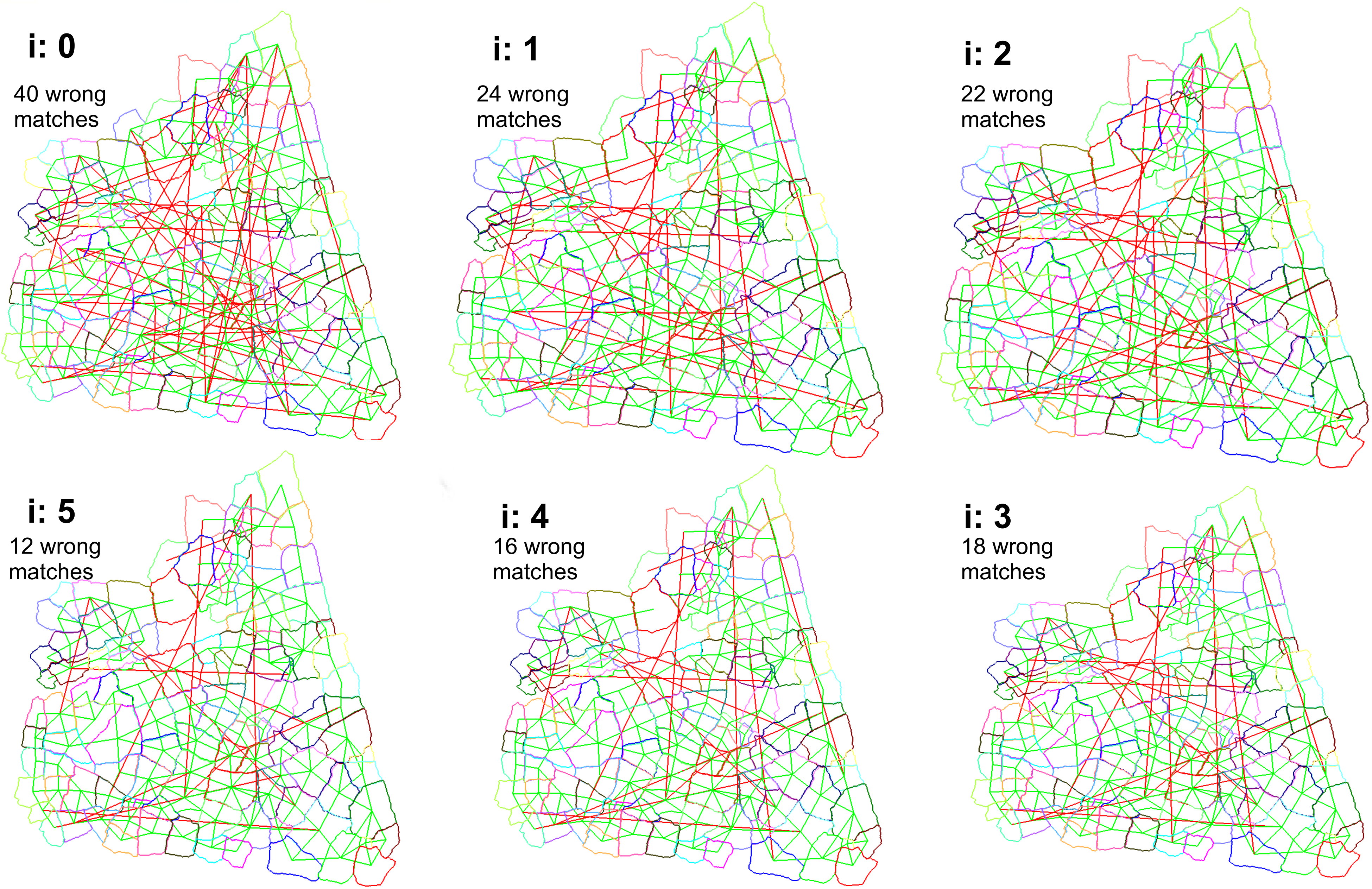
- Contribution of each wrong match:
  - In LS, the contribution is counted towards the residual, hence fails in presence of outliers
  - In nEVM, contributions are steps of a random walk on a unit circle which get mostly cancelled out

Iterative Version (it. n*EVM*)

- nEVM does not discard outliers
- nEVM can be iterated:
  - After each iteration, compare the calculated rotations to the matches
  - If difference is higher than 95th percentile or $\pi/4$, remove the match and repeat
- Provides further improvement of robustness

O. Özyeşil, A. Singer, R. Basri, Stable Camera Motion Estimation using Convex Programming, SIAM Journal on Imaging Sciences.

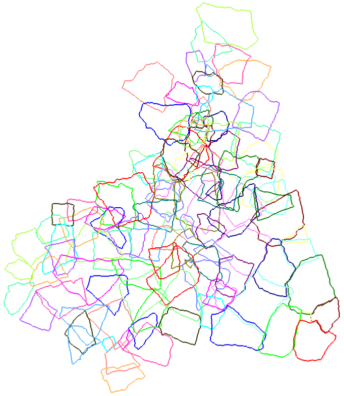
Iterative Version: Example



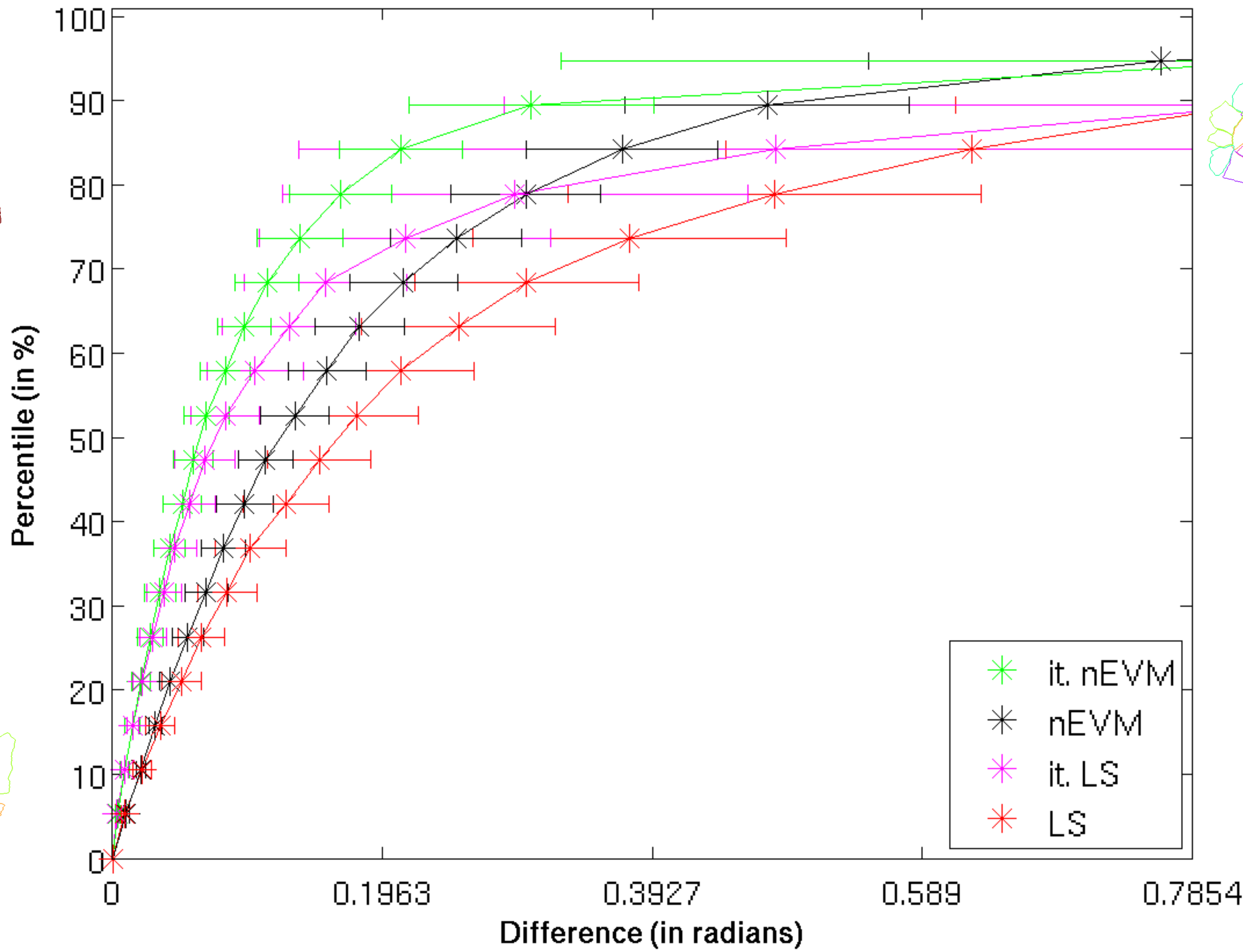
Experimental Results

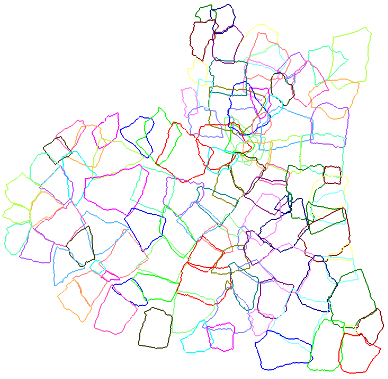
- Set of 239 correct matches that for a connected set of 114 fragments
- 20, 40, or 80 randomly selected high-scoring wrong matches added: controlled percentage of error.
- Test repeated 10 times with different selected sets of wrong matches

40 Wrong Matches



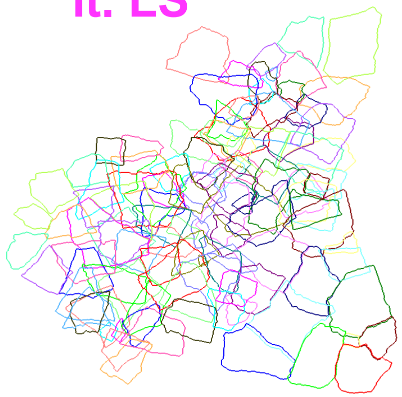
LS



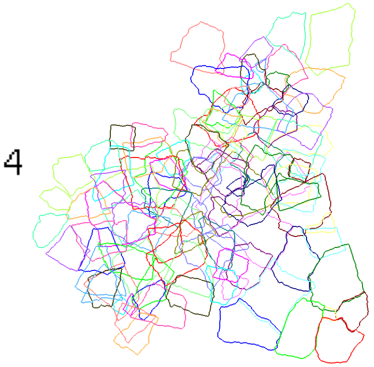


nEVM

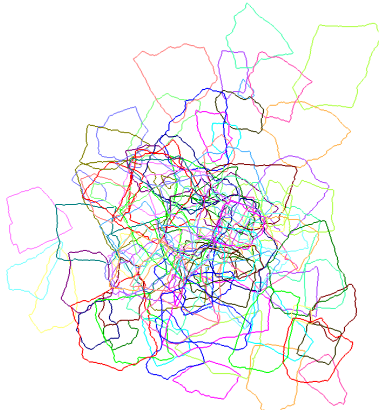
it. LS



it. nEVM

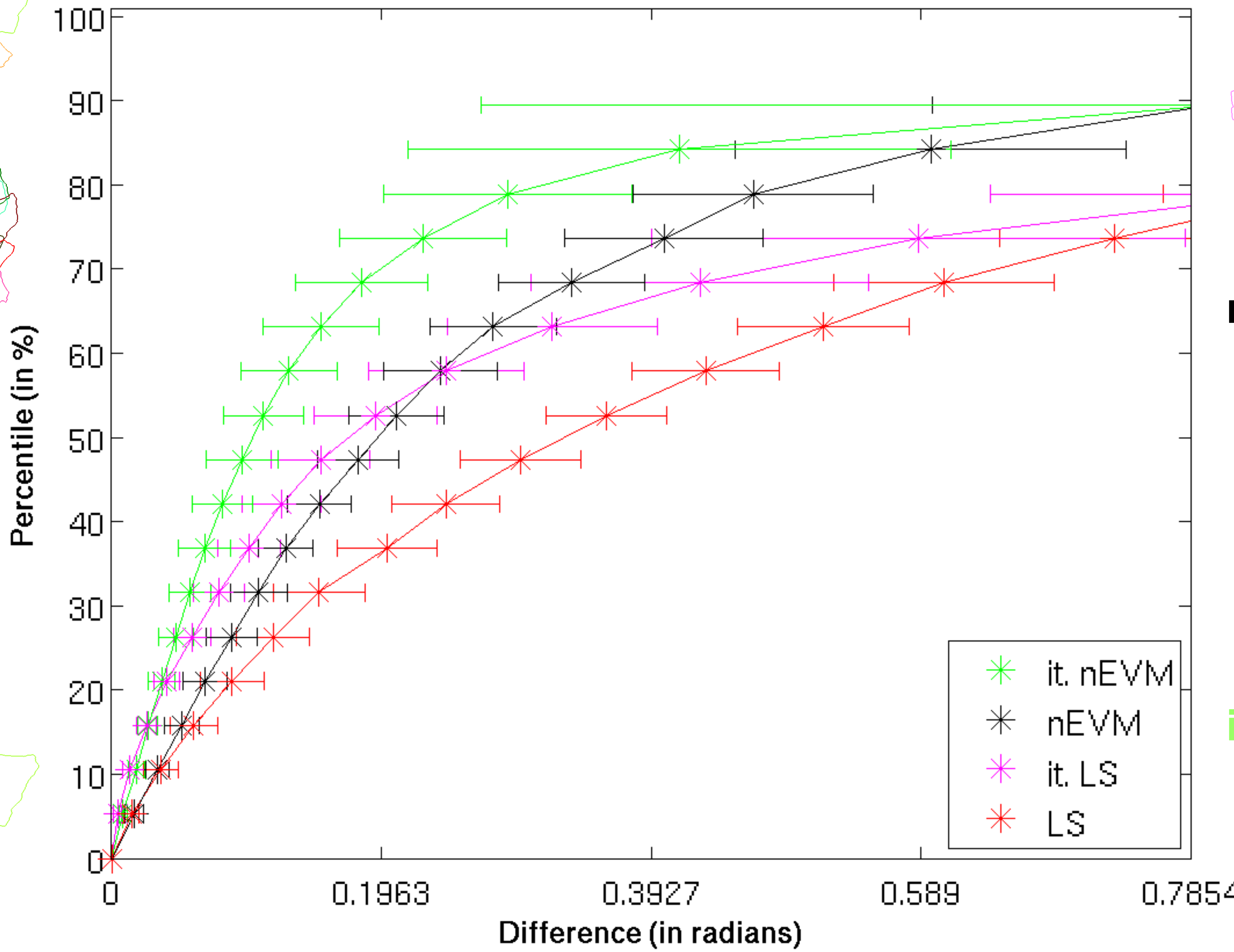


80 Wrong Matches

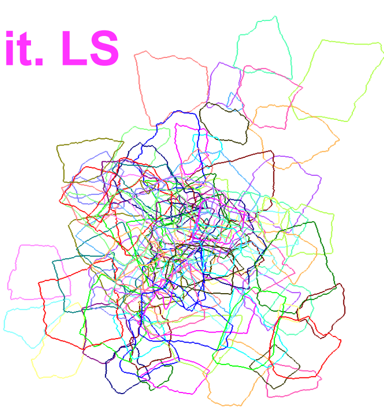


LS

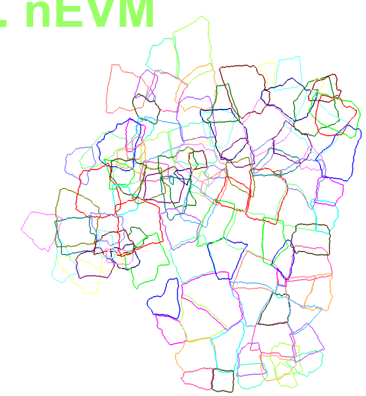
nEVM



it. LS

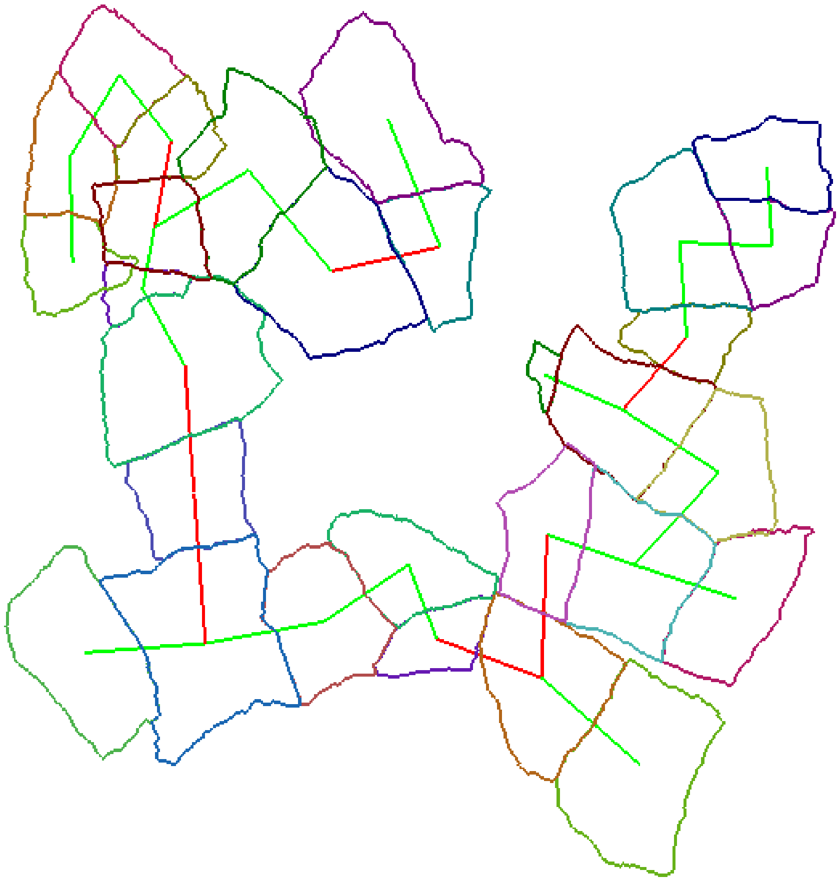


it. nEVM



Limitations of the it. nEVM Method

- The input set of fragments should have redundant paths
- If the graph is sparsely connected, it is difficult to identify outlier matches



Results Summary

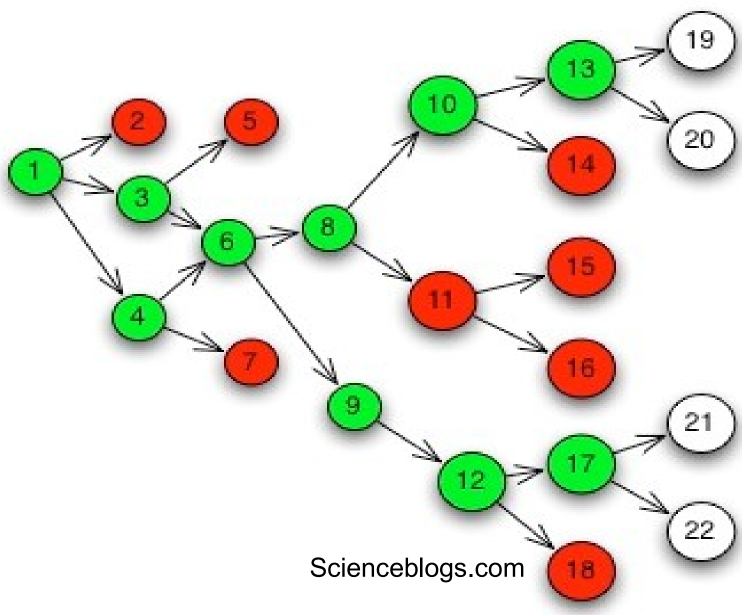
- Normalized angular synchronization methods outperform LS
- Iterative methods detect and undermine outliers better than non-iterative counterparts

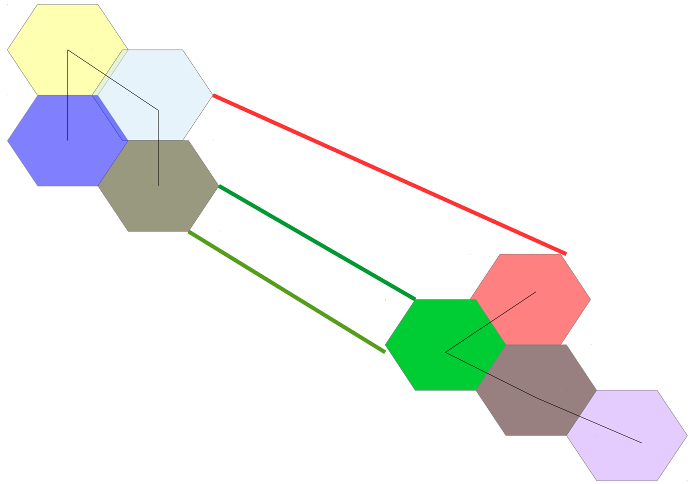
Conclusions and Future Work

- Synchronization helps align fragments robustly to wrong matches
- Connectivity constraints play an important role in its effectiveness

Future Work

- To overcome its limitations, synchronization needs to be incorporated in a search framework that proposes well-connected clusters





Thanks for listening!
Questions?

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